

# A Unified Theory On Some Basic Topological Concepts

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## Abstract

Several mathematicians, including myself, have studied some unifications in general topological spaces as well as in fuzzy topological spaces. For instance in our earlier works, using operations on topological spaces, we have tried to unify some concepts similar to continuity, openness, closedness of functions, compactness, filter convergence, closedness of graphs, countable compactness and Lindelöf property. In this article, to obtain further unifications, we will study  $\varphi_{1,2}$ -compactness and relations between  $\varphi_{1,2}$ -compactness, filters and  $\varphi_{1,2}$ -closure operator.

## 1 . Introduction

Several unifications have been studied in [1, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Some unifications were studied in [10] and [12] by using operations for fuzzy topological spaces. It was claimed there and it can be easily seen that most of the definitions and results can be applied to topological spaces. As far as possible we do not repeat definitions related of known concepts. Because one aim of us to reduce the confusions caused by so much definitions. However, many such definitions will be clear from the special operations considered.

In a topological space  $(X, \tau)$  int, cl, scl etc. will stand for interior, closure, semi-closure operations so on and  $A^\circ$ ,  $\bar{A}$  will stand for the interior of A, the closure of A for a subset A of X respectively.

**Definition 1.1** *Let  $(X, \tau)$  be a topological space. A mapping  $\varphi : P(X) \rightarrow P(X)$  is called an operation on  $(X, \tau)$  if  $A^\circ \subset \varphi(A)$  for all  $A \in P(X)$  and  $\varphi(\emptyset) = \emptyset$ .*

The class of all operations on a topological space  $(X, \tau)$  will be denoted by  $O(X, \tau)$ .

A partial order " $\leq$ " on  $O(X, \tau)$  is defined as  $\varphi_1 \leq \varphi_2 \Leftrightarrow \varphi_1(A) \subset \varphi_2(A)$  for each  $A \in P(X)$ .

An operation  $\varphi \in O(X, \tau)$  is called monotonous if  $\varphi(A) \subset \varphi(B)$  whenever  $A \subset B$  ( $A, B \in P(X)$ ).

**Definition 1.2** Let  $\varphi \in O(X, \tau)$  and  $A, B \subset X$ .  $A$  is called  $\varphi$ -open if  $A \subset \varphi(A)$ .  $B$  is called  $\varphi$ -closed if  $X \setminus B$  is  $\varphi$ -open. Operation  $\tilde{\varphi} \in O(X, \tau)$  is called the dual operation of  $\varphi$  if  $\tilde{\varphi}(A) = X \setminus \varphi(X \setminus A)$  for each  $A \in P(X)$ .

If  $\varphi$  is monotonous, then the family of all  $\varphi$ -open sets is a supratopology ( $\mathcal{U} \subset P(X)$  is a supratopology on  $X$  means that  $\emptyset \in \mathcal{U}$ ,  $X \in \mathcal{U}$  and  $\mathcal{U}$  is closed under arbitrary union [2]).

Let  $(X, \tau)$  be a topological space,  $\varphi \in O(X, \tau)$ ,  $\mathcal{U} \subset P(X)$ ,  $x \in X$ . We use the following notations.

$$\begin{aligned}\mathcal{U}(x) &= \{U : x \in U \in \mathcal{U}\}, \\ \varphi O(X) &= \{U : U \subset X, U \text{ is } \varphi\text{-open}\}, \\ \varphi C(X) &= \{K : K \subset X, K \text{ is } \varphi\text{-closed}\}, \\ \varphi O(X, x) &= \{U : U \in \varphi O(X), x \in U\}.\end{aligned}$$

$$\mathcal{N}(\mathcal{U}, x) = \{N : N \subset X \text{ and there exists a } U \in \mathcal{U}(x) \text{ such that } U \subset N\}.$$

**Definition 1.3** Let  $\varphi \in O(X, \tau)$ ,  $\mathcal{U} \subset P(X)$ .  $\varphi$  is called regular with respect to (shortly w.r.t.)  $\mathcal{U}$  if  $x \in X$  and  $U, V \in \mathcal{U}(x)$ , there exists an  $W \in \mathcal{U}(x)$  such that  $\varphi(W) \subset \varphi(U) \cap \varphi(V)$ .

For any operation  $\varphi \in O(X, \tau)$ ,  $\tau \subset \varphi O(X)$ , and  $X, \emptyset$  are both  $\varphi$ -open and  $\varphi$ -closed.

**Definition 1.4** Let  $\varphi_1, \varphi_2 \in O(X, \tau)$ ,  $A \subset X$ .

a)  $x \in \varphi_{1,2} \text{int} A \Leftrightarrow$  there exists an  $U \in \varphi_1 O(X, x)$  such that  $\varphi_2(U) \subset A$ .

b)  $x \in \varphi_{1,2} \text{cl} A \Leftrightarrow$  for each  $U \in \varphi_1 O(X, x)$ ,  $\varphi_2(U) \cap A \neq \emptyset$ .

c)  $A$  is  $\varphi_{1,2}$ -open  $\Leftrightarrow A \subset \varphi_{1,2} \text{int} A$ .

d)  $A$  is  $\varphi_{1,2}$ -closed  $\Leftrightarrow \varphi_{1,2} \text{cl} A \subset A$ .

If  $A \subset B$  then  $\varphi_{1,2} \text{int} A \subset \varphi_{1,2} \text{int} B$ . Clearly for any set  $A$ ,  $X \setminus \varphi_{1,2} \text{int} A = \varphi_{1,2} \text{cl}(X \setminus A)$  and  $A$  is  $\varphi_{1,2}$ -open iff  $X \setminus A$  is  $\varphi_{1,2}$ -closed.

$\varphi_{1,2} O(X)$  ( $\varphi_{1,2} C(X)$ ) will stand for the family of all  $\varphi_{1,2}$ -open subsets ( $\varphi_{1,2}$ -closed subsets) of  $X$ .

**Theorem 1.5** ([12]). Let  $\varphi_1, \varphi_2 \in O(X, \tau)$ .

a)  $\varphi_{1,2}O(X)$  is a supratopology on  $X$ .

b) If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$  then  $\varphi_{1,2}O(X)$  is a topology on  $X$  and a subset  $K$  of  $X$  is closed w.r.t. this topology iff  $\varphi_{1,2}clK \subset K$ .

c) If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$  and ( $\varphi_2 \geq \iota$  or  $\varphi_2 \geq \varphi_1$ ) then  $\varphi_{1,2}O(X)$  is a topology on  $X$  and a set  $K$  is closed w.r.t. this topology iff  $\varphi_{1,2}clK = K$  (here  $\iota$  is the identity operation).

**Example 1.6** Let the following operations be defined on a topological space  $(X, \tau)$ .

$$\varphi_1 = \text{int}, \quad \varphi_2 = \text{cloint}, \quad \varphi_3 = \text{cl}, \quad \varphi_4 = \text{scl}, \quad \varphi_5 = \iota, \quad \varphi_6 = \text{intocl}.$$

$$\varphi_1 \leq \varphi_2 \leq \varphi_3, \quad \varphi_1 \leq \varphi_5 \leq \varphi_4 \leq \varphi_3, \quad \varphi_1 \leq \varphi_6 \leq \varphi_4.$$

$$\varphi_1O(X) = \tau, \quad \varphi_2O(X) = SO(X) = \text{the family of semi-open sets.}$$

$$\varphi_3O(X) = \varphi_5O(X) = \varphi_4O(X) = P(X) = \text{power set of } X.$$

$$\varphi_6O(X) = PO(X) = \text{the family of pre-open sets.}$$

$$\varphi_{1,3}O(X) = \tau_\theta = \text{the topology of all } \theta\text{-open sets.}$$

$$\varphi_{2,4}O(X) = S\theta O(X) = \text{the family of semi-}\theta\text{-open sets.}$$

$$\varphi_{1,6}O(X) = \tau_s = \text{the semi regularization topology of } X. \text{ It is, the topology with the base } RO(X) \text{ which consists of regular open sets} = \text{the family of } \delta\text{-open sets.}$$

$$\varphi_{2,3}O(X) = \theta SO(X) = \text{the family of all } \theta\text{-semi-open sets.}$$

$\varphi_1, \varphi_3$  ( $\varphi_2, \varphi_6$ ) are the dual operations of each other.  $\varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6$  are regular w.r.t.  $\varphi_1O(X)$ .

$SC(X)$  ( $PC(X)$ ,  $RC(X)$ ,  $S\theta C(X)$ ,  $\theta SC(X)$  respectively) will stand for the family semi-closed (pre-closed, regular closed, semi- $\theta$ -closed,  $\theta$ -semi-closed) sets.

$SR(X) = SO(X) \cap SC(X) = \text{the family of semi-regular sets.}$

For operations  $\varphi_1, \varphi_2 \in O(X, \tau)$ , clearly if  $\varphi_1$  is monotonous and  $\varphi_2 = \iota$  for  $\varphi_1, \varphi_2 \in O(X, \tau)$  then  $\varphi_{1,2}O(X) = \varphi_1O(X)$  and  $\varphi_{1,2}C(X) = \varphi_1C(X)$ . If  $\varphi_1O(X)$  is a topology and  $\varphi_2$  is monotonous then  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$ , so  $\varphi_{1,2}O(X)$  is always a topology.

## 2 . $\varphi_{1,2}$ -closure Operator and Filters

Along of the paper it will be accepted that operations  $\varphi_i, i = 1, 2, \dots$  are defined on a topological space  $(X, \tau)$ .

**Lemma 2.1** If ( $\varphi_2 \geq \varphi_1$  or  $\varphi_2 \geq \iota$ ) then  $\varphi_1O(X) \subset \varphi_2O(X)$ . But the converse is not true.

**Example 2.2** Let  $\varphi_1 = \text{cloint}$ ,  $\varphi_2 = \text{semi-int}$  be defined on  $\mathbb{R}$  with the usual topology. For  $A = (0, 1]$ ,  $\varphi_1(A) = [0, 1]$ ,  $\varphi_2(A) = (0, 1]$  and  $\varphi_1(A) \not\subset \varphi_2(A)$ . i.e.  $\varphi_1 \not\leq \varphi_2$ .

For the set of rational numbers  $\mathbb{Q}$ ,  $\mathbb{Q} \not\subset \varphi_2(\mathbb{Q}) = \emptyset$ . i.e.  $\varphi_2 \not\geq \iota$ . But  $\varphi_1O(\mathbb{R}) = SO(\mathbb{R}) = \varphi_2O(\mathbb{R})$ .

**Theorem 2.3** Let  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1 O(X)\}$ .

a) If  $\varphi_2$  is regular w.r.t.  $\varphi_1 O(X)$  and  $\varphi_1 O(X) \subset \varphi_2 O(X)$  then  $\varphi_{1,2}$ -cl operator defines the same (pre)topology given in Theorem 1.5 (c). Let  $\tau_{\varphi_{1,2}}$  be stand for this topology.  $A \subset \varphi_{1,2} cl A \subset \tau_{\varphi_{1,2}} cl A$  for any subset  $A$  of  $X$ .

b) If  $\varphi_2$  is regular w.r.t.  $\varphi_1 O(X)$ ,  $\varphi_1 O(X) \subset \varphi_2 O(X)$  and  $\mathcal{B} \subset \varphi_{1,2} O(X)$ , then  $\varphi_{1,2}$ -cl operator is a Kuratowski closure operator and  $\varphi_{1,2} cl A = \tau_{\varphi_{1,2}} cl A$  for any subset  $A$  of  $X$ .

**Definition 2.4** ([20]). Let  $\mathcal{F}$  be a filter (or filterbase) in  $(X, \tau)$  and  $a \in X$ .  $\mathcal{F}$  is said to be:

a)  $\varphi_{1,2}$ -accumulates to  $a$  if  $a \in \bigcap \{\varphi_{1,2} cl F : F \in \mathcal{F}\}$ .

b)  $\varphi_{1,2}$ -converges to  $a$  if for each  $U \in \varphi_1 O(X, a)$ , there exists an  $F \in \mathcal{F}$  such that  $F \subset \varphi_2(U)$ .

**Theorem 2.5** ([20]). 1) A filterbase  $\mathcal{F}_b$   $\varphi_{1,2}$ -accumulates ( $\varphi_{1,2}$ -converges) to  $a$  iff filter generated by  $\mathcal{F}_b$   $\varphi_{1,2}$ -accumulates ( $\varphi_{1,2}$ -converges) to  $a$ .

2) If  $\varphi_2$  is monotonous, we can get the family  $\mathcal{N}(\varphi_1 O(X), a)$  instead of  $\varphi_1 O(X, a)$  in the above definitions.

3) A filter  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$  iff  $\{\varphi_2(U) : U \in \varphi_1 O(X, a)\} \subset \mathcal{F}$ .

4) If  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$  then  $\mathcal{F}$   $\varphi_{1,2}$ -accumulates to  $a$ .

5) Let  $\mathcal{F} \subset \mathcal{F}'$  for the filters  $\mathcal{F}$  and  $\mathcal{F}'$

a) If  $\mathcal{F}'$   $\varphi_{1,2}$ -accumulates to  $a$ , then  $\mathcal{F}$   $\varphi_{1,2}$ -accumulates to  $a$ .

b) If  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$  then  $\mathcal{F}'$   $\varphi_{1,2}$ -converges to  $a$ .

6) If  $\varphi_2$  is regular w.r.t.  $\varphi_1 O(X)$  then a filter  $\mathcal{F}$   $\varphi_{1,2}$ -accumulates to  $a \in X$  iff there exists a filter  $\mathcal{F}'$  such that  $\mathcal{F} \subset \mathcal{F}'$  and  $\mathcal{F}'$   $\varphi_{1,2}$ -converges to  $a$ .

7) If  $\mathcal{F}$  is a maximal filter which  $\varphi_{1,2}$ -accumulates to  $a \in X$ , then  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$ .

8) If  $\varphi'_1 O(X) \subset \varphi_1 O(X)$  and  $\varphi'_2 \geq \varphi_2$  for the operations  $\varphi_1, \varphi_2, \varphi'_1, \varphi'_2 \in O(X, \tau)$ , then a filter (or a filterbase)  $\mathcal{F}$   $\varphi'_{1,2}$ -accumulates ( $\varphi'_{1,2}$ -converges) to  $a$  whenever  $\mathcal{F}$   $\varphi_{1,2}$ -accumulates ( $\varphi_{1,2}$ -converges) to  $a$ .

**Proof.** 6) Let  $\varphi_2$  be regular w.r.t.  $\varphi_1 O(X)$  and  $\mathcal{F}$   $\varphi_{1,2}$ -accumulates to  $a$ . Then the family  $\mathcal{F}_b = \{\varphi_2(U) \cap F : U \in \varphi_1 O(X, a), F \in \mathcal{F}\}$  is a filterbase and the filter

$\mathcal{F}'$  generated by  $\mathcal{F}_b$  is finer than  $\mathcal{F}$  and it  $\varphi_{1,2}$ -converges to  $a$ . The other part of the proof is clear.

7) Let  $\mathcal{F}$  be a maximal filter and  $\varphi_{1,2}$ -accumulates to  $a \in X$ . For each  $U \in \varphi_1 O(X, a)$ ,  $\mathcal{F}_{b_u} = \{\varphi_2(U) \cap F : F \in \mathcal{F}\}$  is a filterbase. The filter  $\mathcal{F}_u$  generated by  $\mathcal{F}_{b_u}$  is finer than  $\mathcal{F}$ . So, for each  $U \in \varphi_1 O(X, a)$ ,  $\mathcal{F} = \mathcal{F}_u$ . Now it is clear that  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$ .

A space  $(X, \tau)$  is called  $\varphi_{1,2}$ - $T_2$  if for each  $x, y \in X (x \neq y)$  there are  $\varphi_1$ -open sets  $U_x$  and  $U_y$  such that  $x \in U_x, y \in U_y$  and  $\varphi_2(U_x) \cap \varphi_2(U_y) = \emptyset$  ([12],[20]).

**Theorem 2.6** ([20]) *Let  $(X, \tau)$  be  $\varphi_{1,2}$ - $T_2$  space. If a filter  $\mathcal{F}$   $\varphi_{1,2}$ -converges to some point  $a \in X$  and  $\varphi_{1,2}$ -accumulates to some point  $b \in X$  then  $a = b$ .*

**Proof.** Let's accept that a filter  $\mathcal{F}$  be  $\varphi_{1,2}$ -convergent to  $a$  and  $\varphi_{1,2}$ -accumulate to  $b$  and  $a \neq b$ . Then there exists  $U \in \varphi_1 O(X, a)$  and  $V \in \varphi_1 O(X, b)$  such that  $\varphi_2(U) \cap \varphi_2(V) = \emptyset$ . But  $\varphi_2(U) \in \mathcal{F}$  and  $F \cap \varphi_2(V) \neq \emptyset$  for each  $F \in \mathcal{F}$ . It must be  $\varphi_2(U) \cap \varphi_2(V) \neq \emptyset$ . This contradiction completes the proof.

**Example 2.7** *Let  $a \in X$  and  $\mathcal{F}$  be a filter in  $(X, \tau)$ .*

a) *Let  $\varphi_1 = \text{cloint}, \quad \varphi_2 = \text{cl}$ .*

$\mathcal{F}$   $\varphi_{1,2}$ -converges ( $\varphi_{1,2}$ -accumulates) to  $a$  iff  $\mathcal{F}$   $rc$ -converges ( $rc$ -accumulates) to  $a$  since  $\{\overline{V} : V \in \tau, x \in \overline{V}\} = \{\overline{U} : x \in U \in SO(X)\}$  [8] iff  $\mathcal{F}$   $s$ -converges ( $s$ -accumulates) to  $a$  [5].

b) *Let  $\varphi_1 = \text{int}, \quad \varphi_2 = \text{cl}$ .*

$\mathcal{F}$   $\varphi_{1,2}$ -converges ( $\varphi_{1,2}$ -accumulates) to  $a$  iff  $\mathcal{F}$   $r$ -converges ( $r$ -accumulates) to  $a$  [7] iff  $\mathcal{F}$  almost converges to  $a$  ( $a$  is an almost adherent point of  $\mathcal{F}$ ) [4].

$(X, \tau)$  is  $\varphi_{1,2}$ - $T_2$  iff  $(X, \tau)$  is Urysohn.

c) *Let  $\varphi_1 = \text{int}, \quad \varphi_2 = \text{v}$ .*

$\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$  iff  $\mathcal{F}$  converges to  $a$  in  $(X, \tau)$ .

$(X, \tau)$  is  $\varphi_{1,2}$ - $T_2$  iff  $(X, \tau)$  is Hausdorff.

It is well known that in Hausdorff spaces, a convergent filter can not have more than one accumulation point ([6], page 220).

**Theorem 2.8** ([20]) *Let  $\varphi_1, \varphi_2 \in O(X, \tau)$ .*

1) *If  $\varphi_1 O(X)$  is closed under finite intersection and  $\varphi_1 O(X) \subset \varphi_2 O(X)$ , then for each  $a \in X$ , the family  $\Phi_a = \varphi_1 O(X, a)$  is a filterbase, and  $\Phi_a$   $\varphi_{1,2}$ -converges to  $a$ .*

2) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1 O(X)$  and  $\varphi_1 O(X) \subset \varphi_2 O(X)$ , then for each  $a \in X$  the family  $\Phi_a = \{\varphi_2(U) : U \in \varphi_1 O(X, a)\}$  is a filterbase and  $\varphi_{1,2}$ -converges to  $a$ .*

**Theorem 2.9** ([20]) *Let  $A \subset X$  and  $a \in X$ .*

1) *If there exists a filter which contains  $A$  and  $\varphi_{1,2}$ -accumulates to  $a$ , then  $a \in \varphi_{1,2}clA$ .*

2) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$  and  $a \in \varphi_{1,2}clA$ , then there exists a filter containing  $A$  and  $\varphi_{1,2}$ -converging to  $a$ .*

3) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$ , then  $a \in \varphi_{1,2}clA$  iff there exists a filter  $\mathcal{F}$  containing  $A$  and  $\varphi_{1,2}$ -converging to  $a$ .*

4) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$ , then  $A$  is  $\varphi_{1,2}$ -closed iff whenever there exists a filter containing  $A$  and  $\varphi_{1,2}$ -converging to a point  $a$  in  $X$ , then  $a \in A$ .*

**Proof.** 2) Let  $a \in \varphi_{1,2}clA$ . Then  $\varphi_2(U) \cap A \neq \emptyset$  for each  $U \in \varphi_1O(X, a)$ .

$\Phi = \{\varphi_2(U) \cap A : U \in \varphi_1O(X, a)\}$  is a filterbase. The filter generated by  $\Phi$  contains  $A$  and  $\varphi_{1,2}$ -converges to  $a$ .

The proofs of the others are easy.

**Theorem 2.10** *Let us define  $cl^* : P(X) \rightarrow P(X)$  as  $cl^*A = \{x : \text{there exists a filter } \mathcal{F} \text{ containing } A \text{ and } \varphi_{1,2}\text{-converging to } x\}$  for  $A \subset X$ .*

1) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$  then  $cl^*A = \varphi_{1,2}clA$  and  $cl^*$  operator defines the following (pre)topology.*

$$\tau^* = \{U \subset X : cl^*(X \setminus U) \subset X \setminus U\} = \{U \subset X : \varphi_{1,2}cl(X \setminus U) \subset X \setminus U\} = \tau_{\varphi_{1,2}}.$$

2) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$  and  $\varphi_1O(X) \subset \varphi_2O(X)$  then  $cl^*$  operator defines the following topology.*

$$\tau^* = \{U \subset X : cl^*(X \setminus U) = (X \setminus U)\} = \{U \subset X : \varphi_{1,2}cl(X \setminus U) = X \setminus U\} = \tau_{\varphi_{1,2}}.$$

3) *If  $\varphi_2$  is regular w.r.t.  $\varphi_1O(X)$ ,  $\varphi_1O(X) \subset \varphi_2O(X)$ , and for each  $U \in \varphi_1O(X)$   $\varphi_2(U) \in \varphi_{1,2}O(X)$ , then  $cl^*$  operator is a Kuratowski closure operator and again  $\tau^* = \tau_{\varphi_{1,2}}$ .*

**Example 2.11** *Let  $\varphi_1 = int$ ,  $\varphi_2 = cl$ .*

*A filter  $\mathcal{F}$   $\varphi_{1,2}$ -converges to  $a$  iff  $\mathcal{F}$  is  $\theta$ -converges to  $a$ .  $\theta$ -convergence defines a pretopology [8].*

### 3 . $\varphi_{1,2}$ -compactness

**Definition 3.1** ([20]) *Let  $\varphi_1, \varphi_2 \in O(X, \tau)$ ,  $X \in \mathcal{A} \subset P(X)$ ,  $A \subset X$ .*

a) If  $A \subset \cup \mathcal{U}$  for a subfamily  $\mathcal{U}$  of  $\mathcal{A}$ , then  $\mathcal{U}$  is called an  $\mathcal{A}$ -cover of  $A$ . If an  $\mathcal{A}$ -cover  $\mathcal{U}$  of  $A$  is countable (finite) then we call  $\mathcal{U}$  as countable  $\mathcal{A}$ -cover (finite  $\mathcal{A}$ -cover).

b) If each  $\mathcal{A}$ -cover  $\mathcal{U}$  of  $A$  has a finite subfamily  $\mathcal{U}'$  such that  $A \subset \cup \{\varphi_2(U) : U \in \mathcal{U}'\}$ , then we call  $A$  is  $(\mathcal{A}-\varphi_2)$ -compact relative to  $X$  (shortly  $(\mathcal{A}-\varphi_2)$ -compact set).

c) We call an  $(\mathcal{A}-\varphi_2)$ -compact set relative to  $X$  as  $\mathcal{A}$ -compact set shortly.

d) If we take  $\mathcal{A} = \varphi_1 O(X)$  in (b), then we call  $A$  as  $\varphi_{1,2}$ -compact relative to  $X$  (shortly  $\varphi_{1,2}$ -compact set).

If we take  $\mathcal{A} = \varphi_{1,2} O(X)$  in (c) we get the definition of a  $\varphi_{1,2} O(X)$ -compact set.

If  $X$  is  $\varphi_{1,2}$ -compact set relative to itself, then  $X$  will be called  $\varphi_{1,2}$ -compact space.

If  $X$  is  $\varphi_{1,2} O(X)$ -compact set relative to itself, then  $X$  will be called  $\varphi_{1,2} O(X)$ -compact space.

$\varphi_{1,2}$ -Lindelöf sets relative to  $X$ ,  $\varphi_{1,2}$ -Lindelöf spaces and  $\varphi_{1,2}$ -countable compact sets relative to  $X$ ,  $\varphi_{1,2}$ -countable compact spaces were defined in a similar way as in [21], [22].

**Theorem 3.2** ([20]) *If  $\varphi'_1 O(X) \subset \varphi_1 O(X)$ ,  $\varphi_2 \leq \varphi'_2$  (hence if  $\varphi'_1 \leq \varphi_1, \varphi_2 \leq \varphi'_2$ ), then each  $\varphi_{1,2}$ -compact set is  $\varphi'_{1,2}$ -compact set.*

**Example 3.3** *Let  $A \subset X$ .*

a) *Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{cl}$ .*

*$A$  is  $\varphi_{1,2}$ -compact set iff  $A$  is an  $H$ -set.*

b) *Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{intocl}$ .*

*$A$  is  $\varphi_{1,2}$ -compact set iff  $A$  is an  $N$ -set.*

c) *Let  $\varphi_1 = \text{cloint}$ ,  $\varphi_2 = \text{scl}$ .*

*$A$  is  $\varphi_{1,2}$ -compact set iff  $A$  is an  $s$ -set.*

d) *Let  $\varphi_1 = \text{cloint}$ ,  $\varphi_2 = \text{cl}$ .*

*$A$  is  $\varphi_{1,2}$ -compact set iff  $A$  is an  $S$ -set.*

e) *Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{c}$ .*

*$A$  is  $\varphi_{1,2}$ -compact set iff  $A$  is compact.*

By using Theorem 3.2, we get that, each  $N$ -set is an  $H$ -set, each  $s$ -set is an  $S$ -set and each  $S$ -set is an  $H$ -set.

**Theorem 3.4** ([20]) *The following are equivalent for any subset  $A$  of  $X$ .*

- a)  $A$  is  $\varphi_{1,2}$ -compact set.
- b) Every filterbase in  $X$  which meets  $A$ ,  $\varphi_{1,2}$ -accumulates in  $X$  to some point in  $A$ .
- c) Every maximal filterbase in  $X$  which meets  $A$ ,  $\varphi_{1,2}$ -converges in  $X$  to some point in  $A$ .
- d) Every filterbase in  $A$   $\varphi_{1,2}$ -accumulates in  $X$  to some point in  $A$ .
- e) Every maximal filterbase in  $A$ ,  $\varphi_{1,2}$ -converges in  $X$  to some point in  $A$ .
- f) For any family  $W$  of nonempty sets with  $A \cap (\cap\{\varphi_{1,2}clF : F \in W\}) = \emptyset$ , there exists a finite subfamily  $W'$  of  $W$  such that  $A \cap (\cap W') = \emptyset$ .
- g) For any family of nonempty sets such that for each finite subfamily  $W'$  of  $W$  we have  $A \cap (\cap\{F : F \in W'\}) \neq \emptyset$ , then  $A \cap (\cap\{\varphi_{1,2}clF : F \in W\}) \neq \emptyset$ .
- h) If  $\mathcal{F}$  is a filterbase such that  $A \cap \{\varphi_{1,2}clF : F \in \mathcal{F}\} = \emptyset$  then there exists an  $F \in \mathcal{F}$  such that  $F \cap A = \emptyset$ .

If  $\tilde{\varphi}_2$  is the dual of  $\varphi_2$ , then the following statements (i) and (j) are equivalent to each one of the above statements.

- i) For any family  $\Phi$  of  $\varphi_1$ -closed sets with  $A \cap (\cap\Phi) = \emptyset$ , there exists a finite subfamily  $\Phi'$  of  $\Phi$  such that  $A \cap (\cap\{\tilde{\varphi}_2(F) : F \in \Phi'\}) = \emptyset$ .
- j) If  $\Phi$  is a family of  $\varphi_1$ -closed sets such that for each finite subfamily  $\Phi'$  of  $\Phi$  we have  $A \cap (\cap\{\tilde{\varphi}_2(F) : F \in \Phi'\}) \neq \emptyset$ , then  $A \cap (\cap\Phi) \neq \emptyset$ .

By choosing  $X$  instead of  $A$  in the above Theorem 3.4, we get the equivalent statements for a space  $(X, \tau)$  to be  $\varphi_{1,2}$ -compact space.

**Theorem 3.5** *Let  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1O(X)\}$ .*

- a) *If  $\varphi_2(U) \in \varphi_1O(X)$ ,  $\varphi_2(\varphi_2(U)) \subset \varphi_2(U)$  for each  $U \in \varphi_1O(X)$  then  $\mathcal{B} \subset \varphi_{1,2}O(X) \cap \varphi_1O(X)$ .*
- b) *If  $\varphi_1O(X) \subset \varphi_2O(X)$  and if  $\mathcal{B} \subset \varphi_{1,2}O(X)$ , then  $\mathcal{B}$  is a base for the supratopology  $\varphi_{1,2}O(X)$ .*



c) If  $(\varphi_2 \geq \varphi_1$  or  $\varphi_2 \geq \iota)$  and if  $\mathcal{B} \subset \varphi_{1,2}O(X)$ , then  $\mathcal{B}$  is a base for the supratopology  $\varphi_{1,2}O(X)$  [17].

d) If  $(\varphi_2 \geq \varphi_1$  or  $\varphi_2 \geq \iota)$  and,  $\varphi_2(U) \in \varphi_1O(X)$ ,  $\varphi_2(\varphi_2(U)) \subset \varphi_2(U)$  for each  $U \in \varphi_1O(X)$ , then  $\mathcal{B}$  is a base for the supratopology  $\varphi_{1,2}O(X)$  [18].

**Proof.** a) Let  $U \in \varphi_1O(X)$  and  $x \in \varphi_2(U)$ .  $x \in \varphi_2(U) \in \varphi_1O(X)$  and  $\varphi_2(\varphi_2(U)) \subset \varphi_2(U)$ . So  $x \in \varphi_{1,2}int\varphi_2(U)$ . We have  $\varphi_2(U) \subset \varphi_{1,2}int\varphi_2(U)$ . Hence  $\varphi_2(U) \in \varphi_{1,2}O(X)$  for each  $U \in \varphi_1O(X)$  and  $\mathcal{B} \subset \varphi_{1,2}O(X) \cap \varphi_1O(X)$ .

b) Let  $A \in \varphi_{1,2}O(X)$  and  $x \in A$ . There exists a  $U \in \varphi_1O(X, x)$  such that  $\varphi_2(U) \subset A$ . We have  $x \in U \subset \varphi_2(U) \subset A$ ,  $\varphi_2(U) \in \varphi_{1,2}O(X)$  and  $\varphi_2(U) \in \mathcal{B}$ .

Proofs of (c) and (d) are clear from Lemma 2.1 and (a), (b).

**Theorem 3.6** Let  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1O(X)\}$ . If  $(\varphi_2 \geq \varphi_1$  or  $\varphi_2 \geq \iota)$ , and for each  $U \in \varphi_1O(X)$  we have  $\varphi_2(U) \in \varphi_1O(X)$ ,  $\varphi_2(\varphi_2(U)) \subset \varphi_2(U)$  then the following are equivalent for any subset  $A$  of  $X$ .

a)  $A$  is  $\varphi_{1,2}$ -compact set.

b)  $A$  is  $\mathcal{B}$ -compact set.

c)  $A$  is  $\varphi_{1,2}O(X)$ -compact set.

d) If  $W$  is any subfamily of  $\{X \setminus \varphi_2(U) : U \in \varphi_1O(X)\}$  such that for each finite subfamily  $W'$  of  $W$  we have  $A \cap (\cap W') \neq \emptyset$  then  $A \cap (\cap W) \neq \emptyset$ .

e) If  $W$  is any subfamily of  $\{X \setminus \varphi_2(U) : U \in \varphi_1O(X)\}$  with  $A \cap (\cap W) = \emptyset$  then there exists a finite subfamily  $W'$  of  $W$  such that  $A \cap (\cap W') = \emptyset$ .

f) If  $W$  is any subfamily of  $\{X \setminus U : U \in \varphi_{1,2}O(X)\}$  such that for each finite subfamily  $W'$  of  $W$  we have  $A \cap (\cap W') \neq \emptyset$  then  $A \cap (\cap W) \neq \emptyset$ .

g) If  $W$  is any subfamily of  $\{X \setminus U : U \in \varphi_{1,2}O(X)\}$  such that  $A \cap (\cap W) = \emptyset$ , then there exists a finite subfamily  $W'$  of  $W$  such that  $A \cap (\cap W') = \emptyset$ .

**Proof.** Let us see that  $a \Leftrightarrow c$  is true.

In the case  $A = \emptyset$  proofs are clear.

Let  $A$  be a nonempty  $\varphi_{1,2}$ -compact set and  $\mathcal{U}$  a  $\varphi_{1,2}O(X)$ -cover of  $A$ . For each  $x \in A$ , there exists a  $U_x \in \mathcal{U}$  s.t.  $x \in U_x$ . There exists a  $\varphi_1$ -open set  $V_x$  containing  $x$  s.t.  $V_x \subset \varphi_2(V_x) \subset U_x$ . Since  $A$  is  $\varphi_{1,2}$ -compact set, there exists a finite subset  $\{x_1, \dots, x_n\}$  of  $A$  s.t.  $A \subset \cup_{i=1}^n \varphi_2(V_{x_i}) \subset \cup_{i=1}^n U_{x_i}$ . Hence  $A$  is  $\varphi_{1,2}O(X)$ -compact set.

Let  $A$  be  $\varphi_{1,2}O(X)$ -compact set and  $\mathcal{U}$  a  $\varphi_1O(X)$ -cover of  $A$ . We have  $A \subset \cup\{\varphi_2(U) : U \in \mathcal{U}\}$ . Since for each  $U \in \mathcal{U}$ ,  $\varphi_2(U) \in \mathcal{B} \subset \varphi_{1,2}O(X)$ , there exists a finite subfamily  $\{U_1, \dots, U_n\}$  of  $\mathcal{U}$  s.t.  $A \subset \cup_{i=1}^n \varphi_2(U_i)$ . Hence  $A$  is  $\varphi_{1,2}$ -compact set.

Under the given conditions, since  $\mathcal{B}$  is a base of the supratopology  $\varphi_{1,2}O(X)$ ,  $b \Leftrightarrow c$  is clear. Now the other proofs are easy.

Under the hypothesis of Theorem 3.6 by joining Theorems 3.4 and 3.6 we get the equivalent statements for a set to be  $\varphi_{1,2}$ -compact set.

**Theorem 3.7** *Under the hypothesis of Theorem 3.6, the following are equivalent.*

- a)  $X$  is  $\varphi_{1,2}$ -compact space.
- b)  $X$  is  $\mathcal{B}$ -compact space.
- c)  $X$  is  $\varphi_{1,2}O(X)$ -compact space.
- d) For each  $U \in \varphi_1O(X)$ ,  $X \setminus \varphi_2(U)$  is  $\varphi_{1,2}$ -compact set.
- e) For each  $U \in \varphi_1O(X)$ ,  $X \setminus \varphi_2(U)$  is  $\varphi_{1,2}O(X)$ -compact set.
- f) For each  $U \in \varphi_1O(X)$ ,  $X \setminus \varphi_2(U)$  is  $\mathcal{B}$ -compact set.
- g) Each  $\varphi_{1,2}$ -closed set is  $\varphi_{1,2}$ -compact set.
- h) Each  $\varphi_{1,2}$ -closed set is  $\varphi_{1,2}O(X)$ -compact set.
- i) Each  $\varphi_{1,2}$ -closed set is  $\mathcal{B}$ -compact set.

Now, by using the Theorems 3.4, 3.6 and 3.7, we get the equivalent forms for a space  $(X, \tau)$  to be  $\varphi_{1,2}$ -compact space under the hypothesis of Theorem 3.6.

**Example 3.8** *Let  $A \subset X$ .*

a) Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{intocl}$  as in Example 3.3 (b).  
 $\varphi_1O(X) = \tau$ .  $\varphi_2 \geq \varphi_1$ . For each  $U \in \varphi_1O(X) = \tau$ , we have  $\varphi_2(U) = U^\circ \in \varphi_1O(X)$  and  $\varphi_2(\varphi_2(U)) = (U^\circ)^\circ = U^\circ = \varphi_2(U)$ .  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1O(X)\} = \{U^\circ : U \in \tau\} = RO(X)$ .  $\varphi_{1,2}O(X) = \tau_s$ . We know that  $RO(X)$  is a base for  $\tau_s$ .  $\{X \setminus \varphi_2(U) : U \in \varphi_1O(X)\} = RC(X)$ .  $\varphi_{1,2}C(X)$  is the family of  $\tau_s$ -closed sets is the family of  $\delta$ -closed sets.  $\tilde{\varphi}_2 = \text{cloint}$  is the dual of  $\varphi_2$ .

$A$  is  $\varphi_{1,2}$ -compact set iff it is  $N$ -set iff  $A$  is compact in  $(X, \tau_s)$  iff  $A$  is  $RO(X)$ -compact set.

$(X, \tau)$  is  $\varphi_{1,2}$ -compact iff  $(X, \tau_s)$  is compact. These are very well known results.

b) Let  $\varphi_1 = \text{cloint}$ ,  $\varphi_2 = \text{scl}$  as in Example 3.3 (c).  
 $\varphi_2 \geq \iota$ .  $\varphi_1 O(X) = SO(X)$ ,  $\varphi_{1,2} O(X) = S\theta O(X)$ ,  $\varphi_{1,2} C(X) = S\theta C(X)$ . For each  $U \in \varphi_1 O(X)$  we have  $\varphi_2(U) = \text{scl}U \in SR(X) \subset SO(X)$ , and  $\varphi_2(\varphi_2(U)) = \text{scl}(\text{scl}U) = \text{scl}U = \varphi_2(U)$ .  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1 O(X)\} = SR(X)$  is a base for the supratopology  $S\theta O(X)$ .  $\{X \setminus \varphi_2(U) : U \in \varphi_1 O(X)\} = SR(X)$ .  $\tilde{\varphi}_2 = \text{semi-int}$  is the dual of  $\varphi_2$ .

$A$  is  $\varphi_{1,2}$ -compact set iff it is  $(SO(X)\text{-scl})$ -compact set iff it is  $S\theta O(X)$ -compact set iff it is  $SR(X)$ -compact set.

Now, by using the Theorems 3.4, 3.6 we can write the equivalent statements for a set to be  $(\tau\text{-intocl})$ -compact set, or to be  $(SO(X)\text{-scl})$ -compact set, and by using the Theorems 3.4, 3.6, 3.7 we can write the equivalent statements for a space to be  $(\tau\text{-intocl})$ -compact space or to be  $(SO(X)\text{-scl})$ -compact space.

Some equalities related to closure types can be obtained by using operations and some of them were given in [18]. For example for any open set  $T$  in a topological space  $(X, \tau)$ , we have  $\overline{T} = \theta \text{cl}T = \tau_s\text{-cl}T$ .

**Theorem 3.9** Let  $A \subset X$ . If  $\varphi_2(U) = \varphi_3(U)$  for each  $U \in \varphi_1 O(X)$ , then  $\varphi_{1,2} O(X) = \varphi_{1,3} O(X)$ , and  $A$  is  $\varphi_{1,2}$ -compact ( $\varphi_{1,2} O(X)$ -compact) set iff it is  $\varphi_{1,3}$ -compact ( $\varphi_{1,3} O(X)$ -compact) set.

**Example 3.10** Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{scl}$ ,  $\varphi_3 = \text{intocl}$ .

For  $U \in \varphi_1 O(X) = \tau$ ,  $\varphi_2(U) = \text{scl}U = U \cup U^{\circ} = U^{\circ} = \varphi_3(U)$ , ( $\text{scl}U = U \cup U^{\circ}$ , [3]).  $A$  is  $\tau_s$ -compact set iff  $A$  is  $(\tau\text{-intocl})$ -compact set iff  $A$  is  $RO(X)$ -compact set iff  $A$  is  $(\tau\text{-scl})$ -compact set.

**Theorem 3.11** If  $\varphi_1$  is monotonous and, for each pair  $U, V \in \varphi_1 O(X)$ ,  $\varphi_2(U \cup V) = \varphi_2(U) \cup \varphi_2(V)$ , then the following are equivalent.

a)  $A$  is  $\varphi_{1,2}$ -compact set.

b) Each filterbase  $\mathcal{F}_b$  which is a subfamily of  $\{X \setminus \varphi_2(U) : U \in \varphi_1 O(X)\}$  and meets  $A$ ,  $\varphi_{1,2}$ -accumulates to some point  $a \in A$ .

**Proof.** (a $\Rightarrow$ b). It is clear from Theorem 3.4.

(b $\Rightarrow$ a). Let  $A$  be not  $\varphi_{1,2}$ -compact set under the assumption of (b). There is a subfamily  $\mathcal{U} = \{U_i : i \in I\}$  of  $\varphi_1 O(X)$  s.t.  $A \subset \cup \mathcal{U}$  but for each finite subset  $J$  of  $I$   $A \not\subset \cup_{j \in J} \varphi_2(U_j) = \varphi_2(\cup_{j \in J} U_j)$ . For each finite subset  $J$  of  $I$  we have  $A \cap (X \setminus \varphi_2(\cup_{j \in J} U_j)) \neq \emptyset$ . Since  $\varphi_1$  is monotonous,  $\varphi_1 O(X)$  is a supratopology and hence  $\mathcal{F}_b = \{X \setminus \varphi_2(\cup_{j \in J} U_j) : J \subset I, J \text{ finite}\}$  is a filterbase s.t.  $\mathcal{F}_b \subset \{X \setminus \varphi_2(U) : U \in \varphi_1 O(X)\}$  and  $\mathcal{F}_b$  meets  $A$ . There exists a point  $a$  in  $A$  s.t.  $\mathcal{F}_b$   $\varphi_{1,2}$ -accumulates to  $a$ . There exists a  $U_{i_a} \in \mathcal{U}$  s.t.  $a \in U_{i_a}$ .  $X \setminus \varphi_2(U_{i_a}) \in \mathcal{F}_b$ ,  $\varphi_2(U_{i_a}) \cap (X \setminus \varphi_2(U_{i_a})) \neq \emptyset$ . This contradiction completes the proof.

**Example 3.12** a) Let  $A \subset X$ ,  $\varphi_1 = \text{cloint}$ ,  $\varphi_2 = \text{cl}$  as in Example 3.3 (d).

$\varphi_1 O(X) = SO(X)$ ,  $\varphi_2 \geq \varphi_1$ .  $\varphi_1$  is monotonous. For each  $U \in \varphi_1 O(X) = SO(X)$ ,  $\varphi_2(U) = \bar{U} \in SO(X)$  and  $\varphi_2(\varphi_2(U)) = \varphi_2(U)$ . If  $U, V \in \varphi_1 O(X)$ , then  $\varphi_2(U \cup V) = \overline{U \cup V} = \bar{U} \cup \bar{V} = \varphi_2(U) \cup \varphi_2(V)$ .  $\mathcal{B} = \{\varphi_2(U) : U \in \varphi_1 O(X)\} = RC(X)$ .  $\varphi_{1,2} O(X) = \theta SO(X)$ .

$A$  is  $\varphi_{1,2}$ -compact set iff it is  $\mathcal{B}$ -compact set iff it is  $\varphi_{1,2} O(X)$ -compact set.  $X$  is  $\varphi_{1,2}$ -compact and Hausdorff iff  $X$  is  $S$ -closed space.

$\tilde{\varphi}_2 = \text{int}$  is the dual of  $\varphi_2$ .  $\{X \setminus \varphi_2(U) : U \in \varphi_1 O(X)\} = RO(X) = \{\tilde{\varphi}_2(X \setminus U) : U \in \varphi_1 O(X)\} = \{\text{int}K : K \in SC(X)\}$ .

b) Let  $\varphi_1 = \text{int}$ ,  $\varphi_2 = \text{cl}$  as in Example 3.3 (a).

$\varphi_1$  is monotonous,  $\varphi_2(U \cup V) = \varphi_2(U) \cup \varphi_2(V)$  for  $U, V \in \varphi_1 O(X) = \tau$ .  $\{X \setminus \varphi_2(U) : U \in \varphi_1 O(X)\} = RO(X)$ . For a filterbase  $\mathcal{F}_b \subset RO(X)$ , we have  $\cap\{\varphi_{1,2} \text{cl}F : F \in \mathcal{F}_b\} = \cap\{\theta \text{cl}F : F \in \mathcal{F}_b\} = \cap\{\bar{F} : F \in \mathcal{F}_b\} = \cap\{\tau_s\text{-cl}F : F \in \mathcal{F}_b\}$ .  $X$  is  $\varphi_{1,2}$ -compact and Hausdorff iff  $X$  is  $H$ -closed space.

Now we can get many known results that some of them can be seen from [4,5,7,8] and many unknown results by special choices of operations. And we will have many results by using the papers [18,19,20].

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