HACETTEPE ÜNİVERSİTESİ MATEMATİK BÖLÜMÜ GENEL SEMİNERİ

(HACETTEPE UNIVERSITY MATHEMATICS GENERAL SEMINAR)

Tarih (Date): 26.02.2020, Çarşamba (Wednesday)

Saat (Time): 15:00

Yer (Place): Yaşar Ataman Seminer Salonu

Konuşmacı (Speaker): Ali Devin Sezer, Middle East Technical University, Institute of Applied Mathematics

Başlık(Title): Approximation of Exit Probabilities of Constrained Random Walks

Özet (Abstract): Let X be the constrained random walk on $\mathbb{Z}_+^2$ having increments $(1,0)$, $(-1,1)$, and $(0,-1)$ with probabilities λ, μ_1, and μ_2 representing the lengths of two tandem queues. X is assumed stable and $\mu_1 \neq \mu_2$. Let τ_n be the first time when the sum of the components of X equals n. Let Y be the constrained random walk on $\mathbb{Z} \times \mathbb{Z}_+$ having increments $(-1,0)$, $(1,1)$, and $(0,-1)$ with probabilities λ, μ_1, and μ_2. Let τ be the first time that the components of Y are equal to each other. We prove that $P_{n-x_n(1),x_n(2)}(\tau < \infty)$ approximates $p_{x_n}(\tau_n < \tau_0)$ with relative error *exponentially decaying* in n for $x_n = \lfloor nx \rfloor$, $x \in \mathbb{R}_+^2$, $0 < x(1) + x(2) < 1$, $x(1) > 0$. An affine transformation moving the origin to the point $(n,0)$ and letting $n \to \infty$ connects the X and Y processes. We use a linear combination of basis functions constructed from single and conjugate points on a characteristic surface associated with X to derive a simple expression for $P_y(\tau < \infty)$ in terms of the ratios λ/μ_i. The proof that the relative error decays exponentially in n uses an upper bound on the error probability and a lower bound on p_n obtained via sub and super solutions of a related Hamilton-Jacobi-Bellman equation. We carry out a similar analysis also for the constrained random walk with increments $(1,0)$, $(-1,0)$, $(0,-1)$ and $(0,1)$ representing the lengths of two queues in parallel. Although the main ideas generalize from the tandem case there are also significant differences. We provide a comparison of these two cases.