Feb 2022

Author: Julio D. Rossi (Buenos Aires University)

Title: **Asymptotic Mean Value Properties for the Monge-Ampere Equation**

Abstract: In recent years there has been an increasing interest in whether a mean value property, known to characterize harmonic functions, can be extended in some weak form to solutions of nonlinear equations. This question has been partially motivated by the surprising connection between Random Tug-of-War games and the normalized $p-$Laplacian discovered some years ago, where a nonlinear asymptotic mean value property for solutions of a PDE is related to a dynamic programming principle for an appropriate game.

Our goal in this talk is to show that an asymptotic nonlinear mean value formula holds for the classical Monge-Ampère equation. In particular, we show that u solves

$$\det D^2 u(x) = f(x),$$

if and only if

$$u(x) = \inf_{\substack{\det A = 1 \\ A \leq \varepsilon^{-1/2} Id}} \left\{ \frac{1}{|B_\varepsilon|} \int_{B_\varepsilon(0)} u(x + Ay)\, dy \right\} - \frac{\varepsilon^2\, n}{2(n+2)} \left(f(x)\right)^{1/n} + o(\varepsilon^2)$$

as $\varepsilon \to 0$.

Joint work with P. Blanc (Jyväskylä), F. Charro (Detroit), and J.J. Manfredi (Pittsburgh)