

HACETTEPE UNIVERSITY
DEPARTMENT OF MATHEMATICS
SEMINAR

**CONSTRUCTION OF MODULES WITH A PRESCRIBED
DIRECT SUM DECOMPOSITION**

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Abstract

A ring is said to be semilocal if it semisimple artinian modulo its Jacobson radical. Many classes of interesting modules have a semilocal endomorphism ring: artinian modules [1], linearly compact modules [7] and finitely presented modules over a semilocal ring [3] are between them. I want to explain some methods to construct (finite and infinite) direct sums of modules inside these classes having a prescribed direct sum decomposition and point out some of the problems that appear in order to do that. The case better (but not completely) understood right now is the one of artinian modules which has been recently developed in [4].

There is a general theory describing the behaviour of a finite direct sum of modules with a semilocal endomorphism ring, but not so much is known about infinite direct sums. In this talk I want to explain why trying to understand such infinite direct sums seems a quite challenging and interesting problem.

The main step in the understanding the finite case is the characterization of the commutative monoids that can be realized as the monoid of isomorphism classes of finitely generated projective modules over a semilocal ring [2]. A first step to understand the direct sum decompositions of an infinite direct sums of finitely presented modules over a semilocal ring is to describe the monoid of isomorphism classes of countably generated projective modules over a semilocal ring. The noetherian case was characterized in [5], and it shows that semilocal rings can have a rich supply of countably generated projective modules that are not direct sums of finitely generated ones. The non-noetherian case is still a mystery, but [6] already shows that the general case can be much more pathological than the noetherian one.

REFERENCES

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Place: Yaşar Ataman Meeting Room

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